Unit of Assessment 10 – Mathematical Sciences

Institution	Evidence
Queen Margaret University	As a general point, it has been quite difficult to provide evidence of the various changes described in the questionnaire over a period of 20 years, as to do so would rely on the experience of staff who have been engaged in similar types of research over that 20 year period. This is particularly the case in respect of the first three questions, where the responses are largely qualitative. In relation to the information requested in question 4, this has also proven difficult to access, as typically this detail and type of information is only held for a period of six years. A further general point is that, when looking at subject weightings for research in general, a reasonable sense check for SFC would be to look also at the transparent approach to costing for teaching (TRAC(T)) data that it collects at subject level to inform prices for teaching funding. There will clearly be different costs between research and teaching which means that there would not be a direct comparison, but it might be expected that there would be some correlation, at least in relative terms.
Edinburgh Mathematical Society **(Exact submission also received from the Council for The Mathematical**	The Edinburgh Mathematical Society (EMS) welcomes the opportunity to contribute to this review and to provide evidence relating to the five listed aspects of research activity since FY1998. Note that our quantitative evidence is for the Scottish pre-1992 institutions only and is mainly taken from published submissions for RAE 2001, RAE 2008 and REF 2014. Figures are not available for all of the suggested time points (FY2008 is part-covered by two assessment periods and the last published figures are for FY2013), and we instead use the baseline years FY1998, FY2006 and FY2013 for most of our comparisons. **1. Changes in research practice in areas covered by Unit of Assessment 10.** In recent years there has been an explosion of research activity in areas such as "big data", which involve large

Sciences)	interdisciplinary teams using specialist high performance computational resources to tackle problems of enormous societal and financial impact. Although traditionally associated with individual pen and paper research, mathematical scientists are at the forefront of the big data revolution, and this involves teams of highly-trained researchers and large scale computing infrastructure. The multi-million pound Bayes Centre exemplifies this: not only is it the new home for Scotland's International Centre for Mathematical Sciences (ICMS) it also houses all the Maxwell Institute's first year graduate students, with this co-location designed to facilitate their cohort-based learning (the Maxwell Institute brings together mathematical sciences research activities at Edinburgh and Heriot-Watt). Glasgow has also recently funded a new mathematics and statistics building and St Andrews is in the planning process for a new Mathematics & Statistics building. Boundaries between the mathematical sciences and other disciplines have blurred significantly in the last decade as the result of several changes. These include a growing emphasis on "impact", increases in the availability of computing power and data, and UKRI and EU investment in large and/or challenge-focused grants. Although data science applied to the financial or insurance industries may be the most obvious manifestation of this trend, Scottish mathematicians and statisticians also play leading roles in collaborations with chemists, ecologists, engineers, geo- and marine scientists, medics, physicists and astronomers. Mathematical sciences research in these and many other areas is at a scale unimagined in 1998. It is also important to note that mathematical sciences research is now being carried in many groups located outside traditional university mathematics departments. Centres such as Edinburgh’s Centre for Cognitive Ageing and Cognitive Epidemiology and its Global Academy of Agriculture and Food Security, St Andrews’s Chair of Statistics in Bioscience jointly appointed between the School of Mathematics & Statistics and the Medical School, or in England, Imperial College’s MRC Centre for Outbreak Analysis and Modelling, or Cambridge’s mathematical biology group – located in Plant Sciences – all include mathematical scientists innovating methodology in areas such as bioinformatics or epidemic modelling.

	A similar diaspora of mathematical scientists can be seen in many areas of engineering and informatics. These researchers are typically submitted to UoAs that attract higher levels of REF funding than UoA10, creating a disparity (in Scotland) with the level of funding for analogous research carried out within mathematical sciences departments. It is important to maintain the cohesion of this broad body of mathematical sciences research to ensure the flow of fundamental developments, often initiated within mathematical sciences departments, to groups closer to the applications. **2. Changes in the balance of research activity between constituent discipline areas covered by Unit of Assessment 10.** The Scottish mathematical sciences research landscape has radically transformed since 1997 with an increasing emphasis on interdisciplinary areas such as data analytics (overlapping with UoA 11), life sciences (overlapping with UoAs 1, 5, 7), high performance scientific computation (overlapping with UoAs 9, 12-15) and theoretical physics (overlapping with UoA 9), with all of these overlapping areas costed at the top banding. The UoA 10 REF 2014 impact cases give an indication of the broad range of new activity in mathematical sciences research. These include case studies related to: the use of advanced mathematical modelling techniques to understand tumour growth and design effective therapies (Dundee), ecological models to understand the interactions between red and grey squirrel populations and help to design conservation measures (Maxwell Institute), computational Bayesian methods for inferring infection risk factors and application to control hospital infections (Maxwell Institute); and the development of new statistical techniques for marine environmental monitoring and impact assessment (St Andrews). Applications such as these typically demand powerful computing resources, as well as significant amounts of staff time as mathematical scientists must engage fully with multidisciplinary project teams if impact is to be generated successfully. In actuarial science and financial risk, the Scottish mathematical sciences community has taken the lead in forging links with industry with the establishment in 2010 of the Scottish Financial Risk

	Academy, which pursues a broad portfolio of Knowledge Exchange Activities. Moreover, Heriot- Watt leads the Actuarial Research Centre, a global network for actuarial research, which is attracting funding from industry for fully funded PhD scholarships. Within the Maxwell Institute, the newly awarded EPSRC-funded CDT is emphasising industrial interaction and has support from a wide array of industrial partners for collaborative projects and in St Andrews, applied mathematicians are part of an STFC-CDT in Data Intensive Science. The recent explosion in big data and the importance of machine learning and AI techniques - which together are fuelling a '4th industrial revolution' – sets new challenges with which the mathematical sciences communities must engage. Mathematics and statistics will play central roles in addressing problems relating to e.g. fast and robust learning algorithms, network stability and resilience, and cybersecurity. These recent and ongoing developments signal a shift in the balance towards multidisciplinary aspects of mathematical sciences with increasing demands on computing, and other resources. **3. Changes in levels of support required by academics active in research in the disciplines covered by Unit of Assessment 10, specifically in terms of** **• Research Assistants or equivalents;** **• Specialist support staff such as technicians;** **• Access to specialist research equipment, infrastructure and facilities.** Maintaining and enhancing the high impact of Scottish mathematical sciences research depends crucially on teams of highly trained early-career researchers and high-power computer infrastructure, software and specialist support. Published REF 2014 submissions do not appear to contain RA numbers, and Table 1 lists RA numbers from the RAE 2008 forms RA0 and RA1 (whose census date was 31 Oct 2007) and a current snapshot (from a departmental survey in May 2019). This shows an increase of 60% in RA numbers per category A researcher.

Date	RA total FTE	RAs per cat A FTE
Oct 2007	61.5	0.25
May 2019	94.5	0.40

Table 1. *RA totals and per Category A researcher for Scottish Mathematical Sciences UoAs. Top row: from forms RA0-1 in RAE 2008 (with census date 31 Oct 2007); bottom row: from a departmental survey in May 2019, with Cat A researcher numbers taken as per REF 2014.*

Postgraduate researcher (PGR) numbers for our three baseline years are given in Table 2. The reporting methodology for the two RAEs was the current registered student FTE, but REF 2014 instead asked for the number of research degrees awarded per year.

To enable comparison we have assumed that on average a PGR student takes 4 years between first registration and graduation, but this is likely to be an underestimate. Even so, the table shows a significant increase in PGR numbers (in absolute terms and per Cat A researcher) from 1998 to 2013. The PGR student population has since been further boosted by large awards for a CDT in analysis in 2014 and a CDT in mathematical modelling and collaborative interdisciplinary CDTs in marine science and ultrasonic engineering (all in 2019).

AY End	PGR (FTE)	PGR per CAT A FTE
1998	137.70	0.70
2006	194.09	0.78
2013	249.20*	1.06*

Table 2. *Postgraduate researcher totals and per Category A researcher for Scottish Mathematical Sciences UoAs reported in RAE 2001, RAE 2008 and REF 2014 (*REF 2014 figures are for research degrees awarded per year and we assume an average PhD duration of 4 years)*

Contemporary mathematical sciences research increasingly relies on large and expensive computer equipment and software. For example, Edinburgh has recently spent a six-figure sum on computational infrastructure to facilitate research in tensor analysis and mathematical aspects of machine learning, and St Andrews has recently invested £1M in upgrading its HPC facilities. Both Heriot-Watt and Strathclyde have previously purchased high performance computer (HPC) equipment for advanced scientific computation applications.

The last 10 years have seen an increase in staff dedicated to knowledge exchange (KE) in mathematical sciences. For example, ICMS has employed a full-time Knowledge Transfer Officer since 2010, Edinburgh's School of Mathematics employs a full-time Business Development Executive (BDE) as well as two statistical consultants, and Mathematics & Statistics at Strathclyde has recently appointed a full-time KE fellow. Other institutions provide BDE support for mathematical sciences from central resources. These activities signal an increase in the costs to institutions of supporting KE in the mathematical sciences in the post-Bond era.

4. Changes in the volume (ie number and/or size) of research grants won per researcher active in the disciplines covered by Unit of Assessment 10.

AY end	Research income (£)	Income per cat A FTE (£)
1998	3,573,357	18,230
2006	4,385,090	17,672
2013	9,189,433	39,026

Table 3. *Research income totals and per Category A researcher for Scottish Mathematical Sciences UoAs reported in RAE 2001, RAE 2008 and REF 2014*

Table 3 shows that research income per category A researcher in Scottish mathematical sciences more than doubled from FY2006 to FY2013. This increase cannot be explained by the introduction of fEC to RCUK funding: REF 2014 income figures show that RCUK income made up a decreasing proportion of total mathematical sciences income (from 74% to 65%) during the reporting period (FY2009-13). We also note that EPSRC grant survey data in 2004-05 prior to the introduction of fEC indicated that mathematical sciences research was the most expensive across their portfolio measured by PI time per grant. However, this was unfortunately not reflected in post-fEC funding levels (which gave a uniform 45% uplift and so involved a disproportionate cut in the number of mathematical sciences grants funded).

Evidence indicates that there has been a strong continued growth in external research income, with a substantial number of large grant awards since 2013. Total current funding to PIs in Scottish mathematical sciences departments from EPSRC alone is £31M, with other large awards from NERC and STFC (£3M) and a significant number of European grant awards (e.g. Edinburgh has £5M of funding from the European Commission, St Andrews has over 4M Euro in ERC funding). Assuming that UKRI funding makes up a similar proportion of the total as in FY2013 would give an estimate of £52M total current funding.

5. Any other sources of evidence that might illustrate any changes in the absolute costs of research activity in the disciplines covered by Unit of Assessment 10 since 1997-98.

The change in SFC's research funding allocation mechanism from QR to REG involved a substantial cut to mathematical sciences funding from £7.7M QR awarded in 2008-09 to a REG award of £6.0M in 2009-10. SFC's more recent changes have not altered this and mathematical sciences funding is still far below the 2009-09 award even in flat cash terms (not accounting for inflation). Table 4 shows funding levels for physical science disciplines (the total award and the award per Cat A researcher) from 2008-09 to 2015-16, and the percentage change in total funding over this period is given in Table 5.

UoA	2008-09		2009-10		2015-16	
	QR (£)	QR/V (£)	REG (£)	REG/V (£)	REG (£)	REG/V (£)
Chemistry	7,205,271	45,005	9,639,151	60,513	8,948,568	45,424
Physics	9,561,353	45,227	10,678,647	55,238	11,141,886	56,904
Math Sci	7,701,833	39,904	6,020,472	24,165	6,277,381	26,659
CS & IT	9,042,687	45,647	13,742,023	48,209	10,742,44	34,664

Table 4. *SFC funding (total and total per Cat A researcher) awards to physical science disciplines*

UoA	2009-10	2014-15	2015-16
Chemistry	34%	54%	24%
Physics	12%	19%	17%
Math Sci	-22%	-13%	-18%
CS & IT	52%	63%	19%

Table 5. *% change in total SFC funding to physical science disciplines relative to 2008-09*

The low funding awarded to Scottish mathematical sciences (compared to science and engineering subjects in Scotland and to mathematical sciences in England) is a threat to the health of the discipline.

As shown in Table 6, 5.5% fewer Cat A staff were submitted to REF 2014 than RAE 2008, and it is becoming increasing difficult for departments to make the case for appointments in mathematical sciences in competition with higher-funded areas. This is damaging to future research potential at a time when the economic benefits of mathematical sciences research are becoming more important. For example, two key findings from the 2012 EPSRC-commissioned study (by Deloitte) are that mathematical sciences research contributes 16% of UK gross value added (GVA) and is the basis of 10% of UK jobs.

Cat A staff (FTE)	RAE 2008	REF 2014	% change
Panel B – UK	12,234	13,352	9.1%
Math Sci – UK	1,924	1,931	0.3%
Match-Sci - Scot	249	235	-5.5%

Table 6. *Staff volume submitted to RAE 2008 and REF 2014*

University of St. Andrews	**1. Changes in research practice in areas covered by Unit of Assessment 10.** a) Increasing inter-disciplinarity, often leading to engagement with data-intensive subject areas. b) Hugely increased usage of computation, across all areas of mathematics. c) Areas of mathematics becoming highly data intensive, so costs relating to data management and collection of `big data' becoming significant. d) Changes in costing of research grants (FEC, modernisation of research staff pay). **2 Changes in the balance of research activity between constituent discipline areas covered by Unit of Assessment 10.** Broadly, the main changes within the UoA 10, Mathematical Sciences, at the School of Mathematics and Statistics since 1997-98 have been in the intensity of research activity across the whole School, the increased use of computation across all divisions (Pure, Applied and Statistics), and the associated increased costs of Postdocs and PhD students. A fundamental change in the balance of the School occurred in 1999-2000 when the Schools of Computer Science and Mathematics and Statistics split into two, leading to the founding of the Centre for Interdisciplinary Research in Computational Algebra (CIRCA), housed in the latter School, but bridging research in both. Apart from its purely mathematical uses, software produced by CIRCA has applications to Cryptography, Crystallography and Electronic Engineering. Also in 1998 the group which subsequently became the Vortex Dynamics Group was founded, where much of the focus is on computational fluid models. More recently, the last five years have seen a move into Mathematical Biology, which encompasses cancer modelling, health statistics and computational biology. This has been a significant change in area of mathematical science to a field requiring huge data storage and `big data' analysis. Throughout this period, the Solar and Magnetosphere Theory Group have been at the forefront of the usage of High Performance Computing (HPC) in this research area, where the computational power required to remain at

	the cutting edge has increased hugely (the number of computational gridpoints used in research papers increasing by a factor of approximately 800 between 2001 and 2018). These changes, as well as those in the computational fluid models used by the Vortex Dynamics Group, have led to significantly greater use of computation within Mathematics and Statistics. Increased intensity of research, growth in the UoA globally, increased interdisciplinarity and use of computation have also necessitated growth within the School in terms of manpower and in terms of research support: much of the research carried out involves expensive technical equipment and often large teams of researchers spread across disciplines. **3 Changes in levels of support required by academics active in research in the disciplines covered by Unit of Assessment 10, specifically in terms of** **• Research Assistants or equivalents;** **• Specialist support staff such as technicians;** **• Access to specialist research equipment, infrastructure and facilities.** As above, the changes in the discipline have led to the hugely increased significance of computational power and numbers of staff, Postdoctoral Research Assistants and PhD students. For research staff numbers there were **28.5 FTE in 96**, **31 FTE in 07-08**, and **33.22 FTE in 17-18**. More strikingly, **postdoc numbers have increased by 44%** (**12.83 in 96**, **23.33 in 07-08**, and **18.57 in 17-18**) while PhD numbers have also increased during this period, while the PhD stipend has tripled in this period (**5.5k in 97-98**, **12.6k in 07-08**, **14.6k in 17-18**). These latter numbers demonstrate the increased need for funding for research assistants required in the discipline. In terms of HPC, a cluster which put the School at the forefront of research in Solar and Magnetosphere Theory as well as Vortex Dynamics cost £1/4 million in 2001, while what is essentially a replacement cost £1 million in 2018, an eightfold increase.

For data storage associated to the statistical aspects of cancer research, it is too early to give good data on the levels of support required, but as above, genome sequence data presents huge data storage issues. For the School to make significant impacts in this area, in line with the School strategy, as above petabytes of data will need to be dealt with.

The modernisation of research staff salary scales in 2006, FEC in 2005 and the well-publicised spiralling of journal costs are further factors in the requirement for increased levels of support in the discipline across this period.

4 Changes in the volume (ie number and/or size) of research grants won per researcher active in the disciplines covered by Unit of Assessment 10.

The size of research grants for the School as a whole has remained consistent over the period 1998-2018. For HPC however, the grants have increased significantly. The following is a list of HPC grants awarded to Magnetohydrodynamics (MHD) consortia in which the School was lead applicant:

01/01/99 – 31/12/01	£474,000 JREI Award for MHD Cluster
01/01/03 – 31/12/05	£186,045 PPARC 3D MHD simulations on a parallel computer
01/06/05 - 31/05/07	£238,755 PPARC 3D MHD Simulations on multi-processor, parallel computers
01/04/06 – 31/03/09	£200,000 PPARC 3D MHD Simulations on a parallel Beowulf cluster
01/04/09 - 31/03/12	£945,836 STFC Parallel Computing Resources UK MHD

University of Aberdeen	

Research income	2001 UoA	FTE submitted	1997/98*	2008 UoA	FTE submitted	2006/07**	2014 UoA	FTE submitted	2017/18***
Pure Mathematics	(22)	7	£21,131	(20)	14	£137,368	(10)	25.1	£1,096,067
Applied Mathematics	(23)	1	£1,800	(25)	3	£ 90,259			
Statistical and Operational Research	(24)	4	£33,459						
Total		12	£56,390		17	£227,627		25.1	£1,096,067

*RAE2001 income metrics
**Minor Volume Indicator submission to SFC 2009
***REF2014 income metrics

PDRA/PGR metrics from Minor Volume Indicator Returns 2001 and 2009; Integrated Research Reporting (IRR)

May 2019

1. **Changes in research practice in areas covered by Unit of Assessment 10.**

Unit of Assessment 10 at the University of Aberdeen represents both pure and applied mathematics. While 20 years ago, there was a strong focus on pure mathematics, this has shifted towards applied mathematics, particularly around model-based data analysis and data-based modelling. This kind of research activity requires the use of high-performance computing to support nonlinear time analysis and database modelling.

	In addition, for pure mathematics, since the last REF, there has been growing interest and activity in the area of Applied Topology which also requires access to high performance computing. Research activity in Topology and Algebra has become more intense and PDRFs are required to achieve research outcomes. **2. Changes in the balance of research activity between constituent discipline areas covered by Unit of Assessment 10.** As the figures above illustrate, in the late 1990s the department had a research group in Statistics. This area of activity ceased to exist by the early 2000s. At the same time a new group in Representation Theory was created, changing the research profile and balance. Around 2015 a new group in Applied Topology was created changing the balance of the research more to the Applied Mathematics end of the spectrum. To date, the main areas of activity are in Topology and Algebra (pure). Research in applied mathematics has grown significantly around mathematical modelling and data analysis. Much of this work is multi- and interdisciplinary, translating new abstract results directly into applications in physics, biology, neuroscience, engineering and medicine. The research aims to develop new mathematical approaches to address technological, biological and physical systems and to apply these approaches to develop predictive models. **3. Changes in levels of support required by academics active in research in the disciplines covered by Unit of Assessment 10, specifically in terms of** • Research Assistants or equivalents; • Specialist support staff such as technicians; • Access to specialist research equipment, infrastructure and facilities. Employment of postdoctoral research assistants and fellows has always been part of the research environment within mathematics, and that has remained the case throughout the period for which evidence is sought. For

	pure mathematics, we have also seen a requirement to access high performance computing. Cost recovery and overhead contributions for pure mathematics can be problematic in view of the relatively low external funding levels for grants and contracts for pure mathematics. Modelling of data and the development of predictive models answers many of the Government's funding priorities and is of strategic importance to the institution. It depends on access to high performance and GPU computing facilities and on our ability to invest in such facilities in a sustainable manner and over the longer term. In addition, the institutional high performance computing facility is supported by two high level technicians. Large data sets (1TB-100TB+) require special storage solutions that are integrated into the HPC and GPU clusters and external resources such as Amazon Cloud Servers. The emphasis on partnership working with industry and other non-academic partners further underlines the need for up to date high performance computing facilities and sufficient support to deliver impactful research outcomes. In this respect the resource requirements for mathematics department do not differ substantially from those of many computing science departments. Due to collaborations with national and international partners/clients video conferencing facilities are an important additional resource. **4. Changes in the volume (ie number and/or size) of research grants won per researcher active in the disciplines covered by Unit of Assessment 10.** The interdisciplinary work predominantly undertaken by researchers within Applied Mathematics has provided access to larger and longer grants and contracts than has been the case in the past. This kind of work answers to many of the Government's funding priorities and is of strategic importance to the institution. It depends on access to high performance computing facilities, and on our ability to invest in such facilities in a sustainable manner and over the longer term. Our work has also helped to secure substantial industrial funding; this led to special requirements for computing facilities, regular staff training, data-handling procedures.

	There is a sharp constant increase in average grant income per staff. In the years 1997, 2007, and 2017 the grant income has **increased by a factor of 4 each decade** (**from £56K to £227K to £1M**). The number of staff fluctuated during this period but since 2005 remains at 20-25 staff (pure and applied combined). **5. Any other sources of evidence that might illustrate any changes in the absolute costs of research activity in the disciplines covered by Unit of Assessment 10 since 1997-98.** Compared to 1997-1998, research in collaboration with Industry has grown to >£2.5 million (total over the past five years). This Industry driven research requires access to modern supercomputer facilities including cutting-edge Graphics Processing Units. Software to facilitate such research including Wolfram Mathematica adds to the costs of running simulations and data analysis. Expert statistics software such as SAS will likely increase the software costs further but is essential to be able to consult for Industry in Aberdeen, Scotland, the UK and beyond. As Industry needs to work according to certain regulatory standards, training in these areas for staff is essential and adds again to the costs.
University of Edinburgh	**1. Changes in research practice in areas covered by Unit of Assessment 10.** We have not identified a substantive change in research practice in the areas covered by UoA 10 at the University of Edinburgh. Mathematical Sciences research at Edinburgh has spanned the three discipline areas of Pure Mathematics; Applied Mathematics; and Statistics and Operational Research for the whole period of covered in this review. Within Pure Mathematics, the broad discipline areas have remained as algebra and number theory; analysis and partial differential equations; and geometry and topology throughout the period since 1996. Applied mathematics has seen increasing focus on physical applied mathematics and mathematical physics, including astrophysics, waves and fluids, and also increasing focus on applied analysis and computational mathematics. Statistics and operational research has also retained a consistent breadth of focus between

statistics, operational research and probability.

The research has increased its collaborative focus since 2005, with the launch of the joint Maxwell Institute with Heriot-Watt. Applied Mathematics in particular has a history of interdisciplinary research, with connections to astrophysics research in particular having been in place since at least 2008.

2. **Changes in the balance of research activity between constituent discipline areas covered by Unit of Assessment 10.**

The balance between the different mathematical sciences disciplines has remained approximately consistent between RAE2001 and REF2014 submissions, as shown by the following figures:

Table 1 Staff returned in RAE2001, RAE2008, REF2014

	UoA 20/22 Pure Mathematics	UoA 21/23 Applied Mathematics	UoA 22/24 Statistics and Operational Research	UoA 10 Mathematical Sciences	Total
2000	21	13	10		44
2007	31	17	11		59
2013				57	57

In 2013, the staff submitted spanned the following research groups – A and B indicate Pure Mathematics research; C and D Applied Mathematics; and E and F statistics and operational research:

Table 2 UoA 10 Research Groups submitted to REF2014

	Total staff
A: Algebra, Geometry & Topology	21
B: Analysis & PDEs	10
C: Applied & Computational Mathematics	12
D: Mathematical Physics	4
E: Operational Research	6
F: Probability & Statistics	4
Total	57

3. **Changes in levels of support required by academics active in research in the disciplines covered by Unit of Assessment 10, specifically in terms of**

- **Research Assistants or equivalents;**
- **Specialist support staff such as technicians;**
- **Access to specialist research equipment, infrastructure and facilities.**

The data for research assistants is challenging to compile at this point as we are undergoing a review of which staff should be categorised as research assistants/ 'independent researchers' for REF2021 purposes. As a result, we would expect the number of research assistants and the 'research active' FTEs used in answer to question 4 to change over the next year. It is not possible to be more accurate at this stage.

Table 3 Research assistants

	UoA 20 Pure Mathematics	UoA 22 Applied Mathematics	UoA 23 Statistics and Operational Research	UoA 10 Mathematical Sciences	Total
2000	0	0	1.5		1.5
2007	4	0	3		7
2014				7.57	7.57
2018				21.14	21.14

Table 4 Technical and support staff

	UoA 10
2013	4
2018	4

2000 data is as submitted to RAE2001; 2007 data is as submitted to RAE2008; 2013 and 2018 data are drawn from University of Edinburgh data. Technical staff data is not available for 2000 or 2007.

	4. Changes in the volume (ie number and/or size) of research grants won per researcher active in the disciplines covered by Unit of Assessment 10. ***Table 5 grant income per FTE***

	Total grant income UoA 10 disciplines (£)	FTE	Income per FTE (£)
1997-98	384,349	44	8,375
2006-07	735,193	69	10,655
2012-13	1,483,929	56.8	26,126
2016-17	1,710,482	67.25	25,435

Our post-REF2014 analysis of income is only partially complete – data past 2016-17 is not available at UoA 10 aggregation, and we are aware of grants missing from 2016-17.

University of Dundee	The Mathematics Department in Dundee has a significant research presence in Applied Mathematics. This means that many of our mathematicians use numerical simulations to predict or model processes in other areas of science (biology, physics, medicine, dentistry, etc). The number and size of these simulations have increased notably over the last 20 years. In order to be internationally competitive we now maintain our own Higher Performance Computing (HPC) cluster. This cluster is still small compared with national HPC centres (DiRAC, Archer), but it is necessary to provide computing resources for the day to day experimenting and testing of codes. Once these tests have been successful we then can apply for computing time at national HPC centres for bigger simulations. Increasingly, our cluster is also used to train postgraduate students and provide them with HPC skills which are highly sought after by employers. To regularly update the hardware of such a machine and provide user support is expensive and has to be paid for by the University as it is not covered by UK funding bodies.

	Therefore the expenses we have to provide research infrastructure for applied mathematics have more than doubled over the last 10 years and are now comparable, if not higher, than e.g. that of our computing department.
University of Strathclyde	**1. Changes in research practice in areas covered by Unit of Assessment 10.** Research practice in UoA10 areas has changed considerably since 1997-98 with a corresponding substantial shift in the funding base. The traditional idea of individual research undertaken by mathematicians working with basic tools has long been superseded. Today's mathematicians are involved in highly complex, multidisciplinary projects on a very broad range of topics using large scale, continuously evolving high powered computing infrastructure. Research activity takes place in a context of 'Big Data' and involves mathematical sciences academics working with a range of partners to tackle the global research challenges facing society. The digital revolution means that mathematical science now underpins research in many discipline areas and does so at an unprecedented scale that requires infrastructure that would not have been seen in this research area even in the recent past. Mathematical science has evolved to become a key underpinning component in a wide range of research stretching across numerous UoAs, many of which attract a higher weighting reflecting their well-understood higher cost base. To give just a small number of illustrative examples of current research being undertaken at Strathclyde: • Dr Katy Tant is undertaking large-scale research in collaboration with academic engineering colleagues and industry partners. The aim of her research is to reconstruct maps of the material properties of heterogeneous materials such as those used by the nuclear and aerospace industries. This inverse problem combines sophisticated mathematical models with very large data sets and requires extensive access to high performance computing facilities. • Prof Mike Heath's collaborative research work is being undertaken with many government agencies, research laboratories and academic groupings across Europe and beyond. The aim of his high impact research is to model the ecosystems of our seas and oceans and in particular to advise

	governments on fishing policies. The extremely complex mathematical models are co-created with a wide range of contributors requiring specialised computing environments and technical support. We employ an IT technician to oversee this coding development and also to develop user interfaces for the software that is developed to enable the uptake of the technology by non-technical end users to ensure timely impact on this environmentally and economically important government policy area. **2. Changes in the balance of research activity between constituent discipline areas covered by Unit of Assessment 10.** Mathematical science has developed in such a way that a large element of the research now undertaken by mathematicians is in the application of mathematics in other areas of research. This work requires the development of new and exciting mathematical ideas and concepts but is very much led and guided by the end application. Whilst pure mathematics (or curiosity led research) remains important, the real recent growth and contribution has been in newer areas of application. Researchers are in demand for their mathematical research expertise and techniques to contribute to these applied areas; this also reflects the greater availability of funding available within a context of enhanced career prospects for mathematicians. The areas of Mathematics in which Strathclyde researchers participate by their nature require high performance infrastructure. These areas include: • **Applied Analysis:** mathematical modelling of physical and biological processes leads to complicated sets of differential equations and large networks of interrelated components • **Continuum Mechanics and Industrial Mathematics:** focuses on real-world problems arising in a wide range of different physical contexts, mainly, but not exclusively, using a combination of mathematical and numerical methods to bring new insight into scientific problems arising in continuum mechanics and soft matter modelling. • **Numerical Analysis and Scientific Computing:** construction and analysis of computational methods,

notably for algebraic and differential equations arising in a wide variety application areas.

- **Population Modelling and Epidemiology:** applied in the area of mathematical biology and in the development of population structured models (including participation in MASTS).
- **Stochastic Analysis:** theoretical and methodological problems of various types of stochastic processes in space and time.

The shift towards working in large scale, multidisciplinary teams of researchers to tackle the big problems of today requires the mathematical community to fully understand the complexities of those diverse problems in order to define and deliver the highest impact contribution. Working in these teams (often with a mix of industry, academia and government agencies) requires extremely sophisticated communication skills, state of the art technical skills, and considerable investment in time by academic staff to identify the 3 most effective mechanism for mathematics to contribute, including through the design and construction of the mathematical framework and language to tackle the problem.

Often much of the groundwork requires to be undertaken prior to any funding bid being made and ahead of research teams starting to tackle these research problems. There is a resulting requirement for considerably more input from our most advanced and senior mathematicians in defining the best ways to tackle global strategic challenges.

3. Changes in levels of support required by academics active in research in the disciplines covered by Unit of Assessment 10, specifically in terms of

- **Research Assistants or equivalents;**
- **Specialist support staff such as technicians;**
- **Access to specialist research equipment, infrastructure and facilities.**

As the scale of mathematical science has grown, the infrastructure needs have accelerated. Strathclyde researchers have benefitted from the ARCHIE-WEST high performance computing facility established in March

2012 via a £1.6M award from the EPSRC e-Infrastructure fund to establish a regional centre of excellence in High Performance Computing. The continuing need for this facility is demonstrated in business cases underway to fund growth in storage capacity for the facility.

4. Changes in the volume (ie number and/or size) of research grants won per researcher active in the disciplines covered by Unit of Assessment 10.

	Total Awarded	# Awards	Average per award	Academic FTE	Total Awarded/FTE	# Awards/FTE
1997-98	No equivalent data available from this time period.					
2007-08	£848,466	11	£77,133	37.16	£22,832.77	0.30
2017-18	£1,339,581	12	£111,632	33.4	£40,107.22	0.36

5. Any other sources of evidence that might illustrate any changes in the absolute costs of research activity in the disciplines covered by Unit of Assessment 10 since 1997-98.

The low weighting (and hence relatively lower funding) of UoA10 (compared to science and engineering subjects in Scotland and to mathematical sciences in England) is a major concern in terms of the future health of the discipline, noting the significant shifts in focus of research activity, and associated spend base, highlighted earlier.

In the context of earlier comments regarding the shift to applied research, it is notable that our anticipated staff FTE for submission in UoA10 in REF2021 is likely to be essentially unchanged when compared with REF2014, despite increased demand for the skills and input of our researchers to multidisciplinary research. If we are to ensure ongoing attractiveness of mathematical sciences research it is essential that the increased complexity and funding base of UoA10 academics’ contribution to cross-disciplinary research is recognised through a higher weighting.

Heriot Watt University	Based upon the argument set out below Heriot-Watt University believes that the Research Excellence Grant for UoA 10: Mathematical Sciences should be based on the 1.6 factor, similar to the majority of STEM subjects, rather than the current 1.2 following the REF 2021 outcome. **1. Changes in research practice in areas covered and changes in the balance of activity in Unit of Assessment 10.** There have been significant changes to the mathematical sciences research landscape since 1997. Greater complexity in research problems being addressed by universities and increased government emphasis on challenge led research has resulted mathematicians playing an ever greater role in multidisciplinary teams. Examples of the skills that they bring to these teams include: data analytics, advance solution techniques for partial differential equations, uncertainty analysis and statistical analysis to support risk based approaches to decision making. In fact, these skills are now required across a wide range of research needs, industry applications and decision making processes. Evidence for this can be seen in Heriot-Watt University's REF 14 quality profile (joint with the University of Edinburgh). This submission was ranked 5th in the UK (using the research power metric), 84% of the impact quality profile was rated as 3* or above, with eight of the ten impact case studies being based on the application of excellence in mathematics to the solution of real world problems. **2. Changes in levels of support required by academics active in research in the disciplines covered by Unit of Assessment 10.** As can be seen below the growth in research income via grant has accompanied growth in Research-active Academic Staff numbers and a trend towards Doctoral Training Centres has promoted growth in PhD student numbers. These in turn lead to larger mathematics units incurring greater estates and overhead costs. In addition, many of the applications on mathematics to multidisciplinary problems require access to enhanced computational facilities at both desktop and supercomputing levels. **3. Changes in the volume (ie number and/or size) of research grants won per researcher active in the disciplines covered by Unit of Assessment 10.**

This trend has seen a growth in the number and size of research awards in mathematical sciences, directly to mathematicians as Principal Investigators and as Co-Investigators to promote collaborations with other academic disciplines. In parallel with this growth there has also been a growth in the number of post-graduate research students being supervised within the discipline.

	1997/8	2007/8	2017/18
Research-Active Academic FTE's	29	49	45
Income	259,256	867,800	1,041,000
PGRs	14	35	55

4. Any other sources of evidence that might illustrate any changes in the absolute costs of research activity in the disciplines covered by Unit of Assessment 10 since 1997-98.

At Heriot-Watt University we believe that supporting our mathematicians and facilitating their contribution to multidisciplinary research teams is critical to UK science, its role in achieving the objectives of the Industrial Strategy and in delivering the UN Sustainable Development Goals through enactment of the Global Challenges Research Fund. Our recently launched strategy will:

1. Support our excellence in mathematics with a single academic unit, this is essential as it enables mathematicians to work together on challenging mathematical problems and thereby advance mathematics as a core science.
2. Facilitate their interaction with other areas of our science base to work on multidisciplinary problems, recent examples of success include:

i. Impact of a mathematical model of housing allocation on governmental policies
i. Mathematical modelling that has influenced conservation efforts and policy to prevent squirrel pox spread and the replacement of red squirrels by invasive greys in the UK
i. Statistical methods to control the spread of epidemics

	This approach does however come at a cost which should be recognised through the dual support mechanism, for example: 1. Increasing UKRI grant income in Mathematical Sciences (up to £1.2M in 2019/20) requires an increase in overhead and estates cost from the university. 2. Management and contributions to major profile raising centres such as the EPSRC International Centre for Mathematical Sciences requires financial contributions over and above those of normal research grants. 3. Partnerships in funded Centres for Doctoral Training such as the new Mathematical Modelling, Analysis and Computation CDT require university investment. This secured £5.9M from EPSRC but to ensure success of the submission required a financial commitment of £300k per annum from the participating universities in contributions to staffing and matching studentships. 4. Supporting a growing cohort of self-funding PhD students is critical for reputation and the health of the discipline but requires investment in staff time and student facilities. Overall, these are placing a significant load on the financial viability of mathematics departments. A change from 1.2 to 1.6 in REG weighting would go some way to addressing this.
UKRI	We completed a comparison exercise of grant information for the two financial years 2007/08 and 2017/18. We compared the total value of awards classified against the above UoAs and the cost headings within these awards to identify any changes in costs attributable to defined areas of spend, such as staff costs. These data provided no compelling evidence of significant shifts in costs for any of the above UoAs. Data access restricted our analysis to grants data from 2007 onwards. In addition, the sample size of grants was small for each UoA (e.g. 35 awards for UoA17 in 2007/08) limiting the power of any analysis. Further analysis might identify subtle changes in funding patterns within these UoAs but we do not currently believe there to have been any significant shifts between these periods.

University of Glasgow	**1. Changes in research practice in areas covered by Unit of Assessment 10.**

	Research Costs weights must reflect the actual costs of research, and we welcome the review of these for UoA10. There have been many changes in research practice since these were last reviewed. There has been a move towards greater interdisciplinary research, coupled with a move to larger research grants. For example, a recent large EPSRC grant involves researchers across Mathematics, Statistics and Engineering, as well as clinicians – UoAs with different research cost weightings. Such large grants require dedicated administrative support. Applied mathematicians in UoG are increasingly directly involved in experiments, requiring specialist equipment and infrastructure support, e.g. within UoG: (a) developing tracking tools to track wildebeests from hot-air balloons in sub-saharan Africa; (b) collaboration with the Vet School, via experiments on rabbit hearts. As another example, Big Data and machine learning cuts across Statistics and Computing Science – and nationally, the establishment of places such as the Alan Turing Institute and The Bayes Centre reflects this. This is also reflected in the changing nature of the qualification of staff in statistics in UoG – we have members of staff with PhDs in Physics and Computing Science who now work as professional statisticians (and conversely, there are members of CS who have mathematics PhDs). The use of HPC is not limited to applied mathematical modelling and statistical analysis: increasingly pure mathematicians require access to HPC (e.g. number theorists in UoG, an area we have grown over recent years). All this requires infrastructure support. **2. Changes in the balance of research activity between constituent discipline areas covered by Unit of Assessment 10.**

	The policy of the UoA has been to support a broad research base across pure & applied Mathematics and Statistics. As noted above, the distinctions between "traditional" areas are becoming more blurred (e.g. UoG has research projects connecting applied mathematical modelling and statistics, pure mathematics and ecology, and we are developing link between commercial cyber-security companies and pure mathematicians). **3. Changes in levels of support required by academics active in research in the disciplines covered by Unit of Assessment 10, specifically in terms of** • **Research Assistants or equivalents;** • **Specialist support staff such as technicians;** • **Access to specialist research equipment, infrastructure and facilities.** Technical and administrative research support has not grown in line with increased grant income. **4. Changes in the volume (ie number and/or size) of research grants won per researcher active in the disciplines covered by Unit of Assessment 10.** **5. Any other sources of evidence that might illustrate any changes in the absolute costs of research activity in the disciplines covered by Unit of Assessment 10 since 1997-98.** The value of awards announced in a particular year, as requested in section 4, can be quite uneven due to the total grant award being counted in the year it was awarded. A more informative indicator would be the research income (expenditure) for each year as is reported to HESA. While PhD numbers have grown, this growth has not matched the increase in staff numbers. PhD students in the mathematical sciences are vital to the health of the discipline and are integral to research activities. As outlined in your introductory paragraph, the subject weightings are intended to reflect the varying cost of

	carrying out research in different disciplines and as such, the weightings need to take account of the relative cost of research between disciplines rather than changes within a discipline over time. As detailed above, the cost of research for Mathematical Sciences (UoA 10) has changed significantly over time and has seen an increasing involvement in cross disciplinary research, including direct involvement in experiments requiring specialist equipment and infrastructure support, which we believe is on a par with Computing Science (UoA 11). As a result, the University of Glasgow proposes that the current cost weightings should be increased to reflect the increased costs of undertaking research between the discipline groups.